Next Generation Structural Analysis

Alexander Choi
mitchleonards@yahoo.ca

1. Introduction

For several decades the combination of utility theory and game theory has been the basis of almost every examination of firms, and for rigorous mathematical proof it remains the gold standard. However, the level of abstraction required by this method obscures the technical aspects of the processes being described.

An analytical method that describes the structural attributes of a process and then models it mathematically can counteract this obfuscation, but at the expense of the scientific exactitude that has become the norm in economic literature. While this compromise is unacceptable for some theoretical studies, the relative flexibility and ease of use of a structure-based approach makes it better suited for practical applications.

In the following pages, a structure-oriented approach to analyzing firms is presented, with notes on general accounting.

2. Infrastructure

Given the variety of technologies and resources involved in various industries, it is difficult to construct a compact set of definitions that can describe all of them in detail. For convenience, it is easier to define categories of resources.

Let a factor be defined as a phenomenon with a set of quantitative attributes, a, such that

$$a_t = [a_{1t} \ldots a_{zt}] \tag{2.1}$$

for i = 1 … z attributes at time t.

From this definition, a factor can be a manufacturing plant, a store, a raw material, or a finished product. Since there is no mobility constraint, a factor could also be a transport truck that carries a finished product from a plant to a store. While this blurs the distinction between fixed assets, like buildings, and mobile assets, like transport vehicles, it encapsulates all of the resources involved in a process. The benefits of this flexibility will be shown later.

Since factors can be grouped together, let infrastructure be defined as a set of factors.

3. Processes

Up to this point, nothing has actually been done; the resources involved in a process have simply been defined as factors. Let a process be defined as a set of transitions undergone by a set of factors. Accordingly, factors can be categorized as inputs or outputs.

To be more precise, let an input be defined as a factor that undergoes or facilitates a transition, and let an output be defined as a factor that results from a transition.

The structure of a process can be described using a flow chart, but it can also be useful to have a strictly alphanumeric descriptor. To that end, several further definitions are required.

First, let γ be the identifier of the γth transition in a process, for $\gamma = 1 \ldots \Gamma$ transitions.
Second, let co(γ) be a progression to the γth transition in a process.
Third, let au(γ) be synchronization with the γth transition in a process.
Fourth, let fb(γ) be a regression to the γth transition in a process.

Now, a given process can be described as

$$d = [1, \mathrm{co}(\gamma); \ldots \\ \Gamma, \mathrm{fb}(\gamma)] \tag{3.1}$$

To incorporate the locations where the various transitions take place, let

$$D = [(1,1), \mathrm{co}(\gamma,\zeta); \ldots \\ (\Gamma,Z), \mathrm{fb}(\gamma,\zeta)] \tag{3.2}$$

where ζ is the factor where the γth transition occurs, for $\zeta = 1 \ldots Z$ factors. Since factors can be stationary or mobile, process descriptors can incorporate distribution endogenously.

4. General Accounting

The primary drawback of the process descriptor is that inputs and outputs are not mentioned. While they could be included, the added verbosity would make the descriptor incomprehensible for processes with a significant set of inputs or outputs.

In any case, the process descriptor is only intended as a structural description of a process; for numerical analysis, it is easier to build a mathematical model on top of it rather than force mathematical notation into it.

Accordingly, let α_t be defined as the 1 x A matrix of the set of inputs for a process, such that

$$\alpha_t = [\alpha_{1t} \ldots \alpha_{At}] \tag{4.1}$$

and let δ_t be defined as the A x 1 matrix of prices that corresponds to α_t.

Similarly, let β_{t+n} be the 1 x B matrix of outputs that corresponds to α_t, such that

$$\beta_{t+n} = [\beta_{1t+n} \ldots \beta_{Bt+n}] \tag{4.2}$$

and let ε_{t+n} be defined as the B x 1 matrix of prices that corresponds to β_{t+n}. The time notation here is *t+n* instead of *t* to account for the time difference between the initiation of a process and the time when the outputs are produced.

Using these definitions, the usual equation for profit can be expressed as

$$\pi_{t+n} = \beta_{t+n}\varepsilon_{t+n} - \alpha_t\delta_t \tag{4.3}$$

This equation is somewhat trivial until scale is incorporated.

First, let the relationship between the quantity and price of the inputs be defined as

$$\delta_t = f(\alpha_t) \tag{4.4}$$

Second, let the relationship between the quantity of inputs and the quantity of outputs be defined as

$$\beta_{t+n} = g(\alpha_t, t, n) \tag{4.5}$$

Finally, let the relationship between the quantity and price of the outputs be defined as

$$\varepsilon_{t+n} = h(\beta_{t+n}) \tag{4.6}$$

Incorporating these equations into equation 4.3 gives

$$\begin{aligned} \pi_{t+n} &= g(\alpha_t, t, n)h(\beta_{t+n}) - \alpha_t f(\alpha_t) \\ &= g(\alpha_t, t, n)h(g(\alpha_t, t, n)) - \alpha_t f(\alpha_t) \end{aligned} \tag{4.7}$$

Now, the effect of changes in the set of inputs on profit can be described as

$$d\pi_t/d\alpha_t = g'(\alpha_t, t, n)h(g'(\alpha_t, t, n)) - f'(\alpha_t) \tag{4.8}$$

and the effect of changes in input prices can be described as

$$d\pi_t/d\delta_t = - \alpha_t \tag{4.9}$$

Similar equations for outputs and output prices can also be constructed, but the point of emphasis is that all of the structural aspects of a process are accounted for directly in a single mathematical framework. Assets like manufacturing plants and assembly line equipment can be included in the set of inputs along with raw materials and other components. For outputs, by-products, inventory stockpiles, and sold units can also be grouped together.

The effect of the structural aspects of a process on profits can be calculated using elementary calculus, and where changes in the actual structure of a process are made, a new mathematical model can be easily constructed to describe it.

5. Conclusion

For designing auctions and predicting agent behavior, conventional mathematical methods are incredibly versatile. However, they are difficult to apply to the initial design of business processes. For this task, the structure-oriented model presented here can provide an intuitive means to quickly verify the architecture of a process.

A more technical description of this model is presented in ***Vector Theory (E3)***, which is available on my website (www.geocities.com/mitchleonards).